UCSBTH-97-01
hep-th/9701077

Black Strings and Classical Hair

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Abstract

We examine the geometry near the event horizon of a family of black string solutions with traveling waves. It has previously been shown that the metric is continuous there. Contrary to expectations, we find that the geometry is not smooth, and the horizon becomes singular whenever a wave is present. Both five dimensional and six dimensional black strings are considered with similar results.

January, 1997

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1. Introduction

More than twenty five years ago Wheeler proposed that "black holes have no hair" which captured the idea that in gravitational collapse, most of the information about what forms the black hole is not available classically in the external black hole solution. This was supported by the uniqueness theorem for the Kerr-Newman solution in Einstein-Maxwell theory, and various extensions of this result showing that the solution remains unique when simple additional matter fields are included. It was latter realized that more complicated matter fields can lead to new black hole solutions [1]. But the spirit of the "no hair conjecture" was preserved in that the new solutions contained only a few additional parameters which represented only a small amount of information about the initial state which formed the black hole. For a recent review see [2].

Recently, a potentially more serious violation of the "no hair conjecture" has been proposed [3]. In string theory, one naturally considers a charged black hole which is extended in some internal direction, so the solution describes a black string. It was shown that in the extremal limit, one can add waves to this solution traveling along the string. Unlike most gravitational waves, these waves do not radiate to infinity or fall into the horizon: They appear to be a form of classical hair. Since the waves are not characterized by a few parameters, but instead by arbitrary functions, it was suggested that perhaps all the information about the black string could be contained in these waves on the exterior geometry [3,4].

In [5], the geometry near the event horizon of a black string with traveling waves was examined. It was shown that the metric was at least C^0 there, which was sufficient to insure that the horizon area was well defined. It was expected at the time that the horizon was smooth, and it was just the complicated nature of the solution that made it difficult to find good coordinates everywhere. Here we examine this geometry more closely and show that the curvature actually *diverges* at the horizon whenever the wave is present[1]. We will see that this singularity is null in the sense that all scalar curvature invariants remain finite, and rather mild since the total tidal distortion remains finite. Nevertheless, since the classical "no hair conjecture" usually assumes a regular event horizon, these waves cannot be viewed as examples of classical hair.

There are two different examples of black strings with traveling waves which have been

[1] We assume that the direction along the black string is compact, as required by an "internal" direction. If one relaxes this assumption, some waves are nonsingular.

discussed. One is a six dimensional black string which reduces to a five dimensional black hole, and the other is a five dimensional black string reducing to a four dimensional black hole. There are also different types of waves which can be added to these black strings. In section 2, we examine two different waves on the six dimensional black string and in section 3 we repeat the analysis for the five dimensional black string. In both cases we show that the horizon becomes singular whenever the wave is present. In section 4 we show that the total tidal distortion experienced by an observer falling into the singularity remains finite, which leaves open the possibility that one can pass through this region. Section 5 contains a brief discussion.

2. Curvature of Six Dimensional Black String With Traveling Waves

We start with the low energy effective action of Type IIB string theory in the ten dimensional Einstein frame, keeping only the metric, the dilaton and RR three form H:

$$S = \frac{1}{16\pi G}\int d^{10}x\sqrt{-g}\left(R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{12}e^{\phi}H^2\right) \tag{2.1}$$

A class of extremal black string solutions to (2.1) with traveling waves were studied in [5,6]. The metric for these solutions takes the form:

$$\begin{aligned} ds^2_{10} = &-\left(1-\frac{r_0^2}{r^2}\right)dudv + \left[\frac{p(u)}{r^2} - 2\left(1-\frac{r_0^2}{r^2}\right)\ddot{f}_a(u)y^a\right]du^2 \\ &+\left(1-\frac{r_0^2}{r^2}\right)^{-2}dr^2 + r^2 d\Omega_3^2 + dy_a dy^a \end{aligned} \tag{2.2}$$

where y^a $(a = 1, 2, 3, 4)$ are periodic coordinates on an internal T^4. $u = t - z$ and $v = t + z$ where z is a coordinate on an S^1 and is identified with period L, so $p(u)$ and $f_a(u)$ are periodic functions with period L. The dot represents a derivative with respect to u. When both $p(u)$ and $f_a(u)$ are constant, the metric is stationary and represents the product of T^4 and a six dimensional extremal black string. There is an event horizon at $r = r_0$ which can be shown to be smooth. The functions $p(u)$ and $f_a(u)$ describe waves traveling along this black string, which we will call longitudinal and internal waves respectively.

We begin by considering the case when only the longitudinal wave is present. In this case, the problem is reduced to six dimensions. Let us first compute the curvature in the natural coordinate system, i.e., the coordinates $(u, v, r, \theta, \phi, \psi)$. We will set $r_0 = 1$ for the

rest of this section. Keeping only the longitudinal wave, we can write the the metric as

$$ds_6^2 = -\left(1 - \frac{1}{r^2}\right) dudv + \frac{p(u)}{r^2} du^2 + \left(1 - \frac{1}{r^2}\right)^{-2} dr^2 + r^2 d\Omega_3^2 \tag{2.3}$$

We choose the following tetrad to calculate the curvature components:

$$\begin{aligned}
\omega_1 =& r d\theta \\
\omega_2 =& r \sin\theta d\varphi \\
\omega_3 =& r \sin\theta \sin\varphi d\psi \\
\omega_4 =& \left(1 - \frac{1}{r^2}\right)^{-1} dr \\
\omega_5 =& \frac{p^{1/2}(u)}{r} du - \frac{r^2 - 1}{2rp^{1/2}(u)} dv \\
\omega_6 =& \frac{r^2 - 1}{2rp^{1/2}(u)} dv
\end{aligned} \tag{2.4}$$

so $ds^2 = \omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2 + \omega_5^2 - \omega_6^2$. In this tetrad, the curvature components turn out to be

$$\begin{aligned}
R_{1212} = R_{2323} = R_{3131} &= \frac{2r^2 - 1}{r^6} \\
R_{1414} = R_{2424} = R_{3434} &= \frac{-2r^2 + 2}{r^6} \\
R_{1515} = R_{2525} = R_{3535} &= \frac{(r^2 - 1)^2}{r^6} \\
R_{1616} = R_{2626} = R_{3636} &= \frac{r^4 - 1}{r^6} \\
R_{1516} = R_{2526} = R_{3536} &= \frac{r^2 - 1}{r^4} \\
R_{4545} &= \frac{-3r^4 + 6r^2 - 4}{r^6} \\
R_{4546} &= \frac{-3r^2 + 3}{r^4} \\
R_{4646} &= \frac{-3r^4 + 4}{r^6} \\
R_{5656} &= \frac{1}{r^6}
\end{aligned} \tag{2.5}$$

The rest of the components are zero except the ones that can be obtained by symmetry. Notice that all curvature components are finite at the horizon $r = 1$. In fact, they are all independent of $p(u)$! This is a special property of the tetrad (2.4) that we have chosen.

But it follows that all scalar curvature invariants are independent of $p(u)$ and finite at the horizon. The Ricci tensor takes a very simple form:

$$\begin{aligned} R_{11} &= R_{22} = R_{33} = \frac{2}{r^6} \\ R_{44} &= -\frac{2}{r^6} \\ R_{55} &= -\frac{2}{r^6} \\ R_{66} &= \frac{2}{r^6} \end{aligned} \tag{2.6}$$

At first sight, it appears that these results show that the curvature is completely well behaved as one approaches the horizon. But one must realize that the (future) event horizon lies not only at $r = 1$, but also at $u, v = +\infty$ and the coordinate system (u, v, r) becomes ill-defined at the horizon. More importantly, the tetrad (2.4) also becomes ill-defined at the horizon and is related to a good tetrad by an infinite boost involving $\omega_4, \omega_5, \omega_6$. The black string curvature components (2.5) are not invariant under a general boost, so potential divergences may arise. (It is interesting to notice that the $4, 5, 6$ components of the Ricci tensor (2.6) are proportional to the three dimensional Minkowski metric and hence are boost invariant. This is consistent with the fact that the Ricci tensor does not change when a traveling wave is added [7].) For the $p(u) = \textit{constant}$ case (no traveling wave), an infinite boost is still necessary, but turns out not to cause a problem. An analytic extension through the horizon has been found in the Appendix of [5] and the horizon is smooth. For the general case, $p(u)$ oscillates an infinite number of times before reaching the horizon and this new feature causes the curvature singularity at the horizon, as we will see.

A coordinate system in which the black string metric (2.2) is C^0 at the horizon was found in [5], although only the leading order terms near the horizon were given explicitly. The exact metric (for the case of a longitudinal wave) can be written as

$$\begin{aligned} ds^2 = &- W^2 dU dV + \sigma^2 W^4 \left[(r^2-1)(4r^4-3)\right] dU^2 \\ &+ \left[2\sigma W^4 r^2 (r^2-1)(2r^2+1) + 6W^2 \left(\int_0^U \sigma^2 W^4 dU\right)\right] dq dU \\ &+ W^4 r^6 dq^2 + r^2 d\Omega_3^2 \end{aligned} \tag{2.7}$$

where the coordinate transformations are given by

$$
\begin{aligned}
&\sigma^2(u) + \dot{\sigma}(u) = p(u) \\
&G(u) = e^{\int_0^u \sigma du} \\
&U = -\int_u^{+\infty} \frac{du}{G^2} \\
&W = G\left(1 - \frac{1}{r^2}\right)^{1/2} \\
&q = -\frac{1}{2W^2} - 3\int_0^U \sigma dU \\
&V = v - \frac{\sigma(u)}{r^2 - 1} - 2\int_0^u \sigma^2 du + 3\int_0^U \sigma^2 W^2 dU
\end{aligned}
\tag{2.8}
$$

Some motivation for these transformations can be found in [5] (where V was called ν). $\sigma(u)$ is a periodic function with the same period L as $p(u)$. U is a function of u and is one of the new coordinates. The (future) event horizon lies at $U = 0$ and $U < 0$ corresponds to the region outside the horizon. (The above definition of U only holds outside the horizon. Inside the horizon, U is defined to be $U = \int_{-\infty}^{u} \frac{du}{G^2}$.) r and W are functions of U and q and can be obtained explicitly by inverting (2.8). It follows that $r = 1$ when $U = 0$. The metric is independent of V, so $\partial/\partial V$ is a null Killing vector. The last term in the definition of V is an integral over U at constant q. It contributes $3\sigma^2 W^2 dU + 6(\int_0^U \sigma^2 W^4 dU)dq$ to dV.

The horizon area is $A = 2\pi^2 \int_0^L \sigma du$ [5]. Notice that we can actually choose different wave profiles $p_{in}(u), p_{out}(u)$ for the region inside and outside the horizon as long as they satisfy a matching condition:

$$
\int_0^L \sigma_{in}(u) du = \int_0^L \sigma_{out}(u) du \tag{2.9}
$$

i.e., the horizon area has to match.

The metric (2.7) is only C^0 at $U = 0$ since σ is a periodic function of u and $U = 0$ corresponds to $u = \infty$. So σ oscillates an infinite number of times near the horizon and does not have a well defined limit. For simplicity, we will choose the following null tetrad

to calculate the curvature components

$$
\begin{aligned}
\omega_1 =& r d\theta \\
\omega_2 =& r \sin\theta d\varphi \\
\omega_3 =& r \sin\theta \sin\varphi d\psi \\
\omega_4 =& W^2 r^3 dq \\
\omega_5 =& W^2 dU \\
\omega_6 =& -\frac{1}{2}dV + \frac{1}{2}\sigma^2 W^2 \left[(r^2-1)(4r^4-3)\right] dU \\
&+ \left[\sigma W^2 r^2 (r^2-1)(2r^2+1) + 3\left(\int_0^U \sigma^2 W^4 dU\right)\right] dq
\end{aligned}
\tag{2.10}
$$

so $ds^2 = \omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2 + 2\omega_5\omega_6$. This is a good tetrad for the entire region, including the horizon. The curvature components turn out to be

$$
\begin{aligned}
R_{1212} &= R_{2323} = R_{3131} = \frac{2r^2-1}{r^6} \\
R_{1414} &= R_{2424} = R_{3434} = \frac{-2r^2+2}{r^6} \\
R_{1415} &= R_{2425} = R_{3435} = \frac{-2r^2+3}{r^3}\sigma \\
R_{1515} &= R_{2525} = R_{3535} = \frac{\dot{\sigma}}{r^2-1} + O(\dot{\sigma},\sigma) \\
R_{1516} &= R_{2526} = R_{3536} = \frac{-r^2+1}{r^6} \\
R_{4545} &= -\frac{3\dot{\sigma}}{r^2-1} + O(\dot{\sigma},\sigma) \\
R_{4546} &= \frac{3r^2-4}{r^6} \\
R_{4556} &= \frac{6r^2-9}{r^3}\sigma \\
R_{5656} &= \frac{1}{r^6}
\end{aligned}
\tag{2.11}
$$

The rest of the components are zero except the ones that can be obtained by symmetry. The above expressions are valid everywhere, with $O(\dot{\sigma},\sigma)$ denoting terms which are finite at the horizon ($r=1$). The components R_{1515} , R_{2525} , R_{3535} , R_{4545} clearly diverge at the horizon and the geometry is singular. An observer crossing the horizon would feel infinite tidal force. The curvature diverges when $\lim_{u\to\infty}\dot{\sigma}(u) \neq 0$. This will hold whenever $p(u)$

is not constant, since it is periodic and σ is defined as in (2.8). This calculation of the curvature is reliable even though the metric is only C^0 at the horizon, for the following reason. The metric is analytic everywhere away from the horizon and in this region we can calculate the curvature components in either tetrad (2.4) or tetrad (2.10). As one approaches the horizon, one physically wants to compute the curvature in a frame which is parallelly propagated along a geodesic. When checking for curvature singularities, it suffices to use a tetrad which is continuous across the horizon. The C^0 metric is sufficient for determining continuous tetrads. Calculating the curvature components in tetrad (2.10) is equivalent to boosting the curvature components calculated in (2.4) and an infinite boost is needed as we approach the horizon.

We have also calculated the curvature when both longitudinal and internal waves are present. The result is very similar. In the continuous tetrad, the divergent components are of the form $\frac{\dot{\sigma}(u)}{r^2-1}$ and $\frac{\ddot{f}_a(u)}{r^2-1}$. As before, all scalars remain finite at the horizon.

There are two other types of traveling waves that were discussed in [5] and [6]. They represent the oscillation of the black string in the four external spatial directions and the angular momentum of the black string. We did not calculate the curvature for these two types of waves but we expect the result to be similar.

3. Curvature Of Five Dimensional Black String With Traveling Waves

Type IIA string theory admits a class of solutions describing five dimensional extremal black string with traveling waves. The metric takes the form

$$\begin{aligned} ds_{10}^2 = & -\left(1-\frac{r_0}{r}\right) dudv + \left[\frac{p(u)}{r} - 2\left(1-\frac{r_0}{r}\right)\ddot{f}_a(u)y^a\right] du^2 \\ & + \left(1-\frac{r_0}{r}\right)^{-2} dr^2 + r^2 d\Omega^2 + dy_a dy^a \end{aligned} \tag{3.1}$$

The coordinates are similar to the six dimensional case except that there are now five periodic y^a coordinates parameterizing an internal T^5. As before, we refer to the waves described by $p(u)$ and $\ddot{f}_a(u)$ as longitudinal and internal waves.

All the results of the five dimensional black string will be very similar to the previous section. We first consider the case where only the longitudinal wave exists and comment on other types of waves later.

When there is only a longitudinal wave, the problem is reduced to five dimensions. Let us first compute the curvature in the natural coordinate system (u, v, r, θ, ϕ). Keeping

only the longitudinal wave and setting $r_0 = 1$, we can write the metric as

$$ds_5^2 = -\left(1 - \frac{1}{r}\right) dudv + \frac{p(u)}{r} du^2 + \left(1 - \frac{1}{r}\right)^{-2} dr^2 + r^2 d\Omega^2 \tag{3.2}$$

We choose the following tetrad to calculate the curvature components:

$$\begin{aligned}
\omega_1 =& r d\theta \\
\omega_2 =& r \sin\theta d\varphi \\
\omega_3 =& \left(1 - \frac{1}{r}\right)^{-1} dr \\
\omega_4 =& \left[\frac{p(u)}{r}\right]^{1/2} du - \frac{r-1}{2[p(u)r]^{1/2}} dv \\
\omega_5 =& \frac{r-1}{2[p(u)r]^{1/2}} dv
\end{aligned} \tag{3.3}$$

so $ds^2 = \omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2 - \omega_5^2$. In this tetrad, the nontrivial curvature components are

$$\begin{aligned}
R_{1212} &= \frac{2r-1}{r^4} \\
R_{1313} &= R_{2323} = \frac{-r+1}{r^4} \\
R_{1414} &= R_{2424} = \frac{(r-1)^2}{2r^4} \\
R_{1415} &= R_{2425} = \frac{r-1}{2r^3} \\
R_{1515} &= R_{2525} = \frac{r^2-1}{2r^4} \\
R_{3434} &= \frac{-4r^2+8r-5}{4r^4} \\
R_{3435} &= \frac{-r+1}{r^3} \\
R_{3535} &= \frac{-4r^2+5}{4r^4} \\
R_{4545} &= \frac{1}{4r^4}
\end{aligned} \tag{3.4}$$

Once again, all curvature components are independent of $p(u)$ and all scalars that can be formed out of curvature tensor and the metric are finite at horizon. The Ricci tensor

takes a simple form

$$\begin{aligned}
R_{11} =& \frac{1}{r^4} \\
R_{22} =& \frac{1}{r^4} \\
R_{33} =& -\frac{1}{2r^4} \\
R_{44} =& -\frac{1}{2r^4} \\
R_{55} =& \frac{1}{2r^4}
\end{aligned} \tag{3.5}$$

All curvature components seems to be well behaved at the horizon. However, similar to the six dimensional case, the (future) event horizon lies at $r = 1,\ u, v = +\infty$ and the tetrad (3.3) becomes ill-defined at the horizon. It is again related to a good tetrad by an infinite boost. For the $p(u) = constant$ case, an analytic extension through the horizon can be found. For the general case, $p(u)$ oscillates an infinite number of times before reaching the horizon and this causes a curvature singularity there.

By copying the steps in the six dimensional black string case, we can find new coordinates in which the metric is C^0 at the horizon. The metric then takes the form

$$\begin{aligned}
ds^2 =& -\frac{2W^2}{r} dU dV + 4\sigma^2 W^4 \frac{(r-1)(9r^2+3r-2)}{r} dU^2 \\
&+ \left(8\sigma W^4 \frac{(r-1)(3r^2+2r+1)}{r} + \frac{48W^2}{r} \int_0^U \sigma^2 W^4 dU \right) dU dq + 4W^4 r^2 dq^2 + r^2 d\Omega^2
\end{aligned} \tag{3.6}$$

where the coordinate transformations are given by

$$\begin{aligned}
&\sigma^2(u) + 2\dot{\sigma}(u) = p(u) \\
&G(u) = e^{\frac{1}{2}\int_0^u \sigma(u) du} \\
&U = -\frac{1}{2}\int_u^{+\infty} \frac{du}{G^2} \\
&W = G(r-1)^{1/2} \\
&q = -\frac{1}{2W^2} - 3\int_0^U \sigma dU \\
&V = v - \frac{2\sigma}{r-1} - 3\int_0^u \sigma^2 du + 12\int_0^U \sigma^2 W^2 dU
\end{aligned} \tag{3.7}$$

$\sigma(u)$ is a periodic function with period L. The event horizon lies at $U = 0$ and $U < 0$ corresponds to the region outside the horizon. As in the six dimensional case, we can

choose different wave profiles $p_{in}(u), p_{out}(u)$ for regions inside and outside the horizon as long as the horizon area is matched. The metric is analytic both inside and outside the horizon but is only continuous at the horizon. We will calculate the curvature components in the null tetrad

$$
\begin{aligned}
\omega_1 =& r d\theta \\
\omega_2 =& r \sin\theta d\varphi \\
\omega_3 =& 2W^2 r dq \\
\omega_4 =& \frac{2W^2}{r} dU \\
\omega_5 =& -\frac{1}{2} dV + \sigma^2 W^2 (r-1)(9r^2+3r-2) dU \\
& + \left[2\sigma W^2 (r-1)(3r^2+2r+1) + 12 \left(\int_0^U \sigma^2 W^4 dU \right) \right] dq
\end{aligned}
\tag{3.8}
$$

so $ds^2 = \omega_1^2 + \omega_2^2 + \omega_3^2 + 2\omega_4\omega_5$. The nontrivial components of the curvature are

$$
\begin{aligned}
R_{1212} &= \frac{2r-1}{r^4} \\
R_{1313} &= R_{2323} = \frac{-r+1}{r^4} \\
R_{1314} &= R_{2324} = \frac{-3r+4}{2r^2}\sigma \\
R_{1414} &= R_{2424} = \frac{\dot\sigma}{r-1} + O(\sigma, \dot\sigma) \\
R_{1415} &= R_{2425} = \frac{-r+1}{2r^4} \\
R_{3434} &= -\frac{2\dot\sigma}{r-1} + O(\sigma, \dot\sigma) \\
R_{3435} &= \frac{4r-5}{4r^4} \\
R_{3445} &= \frac{3r-4}{r^2}\sigma \\
R_{4545} &= \frac{1}{4r^4}
\end{aligned}
\tag{3.9}
$$

The components R_{1414}, R_{2424} and R_{3434} clearly diverge at the horizon $r = 1$. We also calculated the curvature in the case where both longitudinal and internal waves are present. The divergent components are of the form $\frac{\dot\sigma(u)}{r-1}$ and $\frac{\ddot f_a(u)}{r-1}$ and all scalars remain finite at the horizon.

There is another type of traveling wave that was discussed in [6]. It represents the oscillation of the black string in the three external spatial directions. We did not calculate the curvature for this case but we expect the result to be similar.

4. The Curvature Components Are Integrably Finite At The Event Horizon

A free falling object feels a tidal force which is proportional to the curvature components $R_{i\tau j\tau}$, where τ, i ($i = 1, 2, ..., D-1$) denote components in an orthonormal frame carried by the observer. For the black string solutions with traveling waves discussed in this paper, these components diverge at the horizon. However in this section we show that the curvature components have the property that they are integrably finite in the sense that for a free-falling observer, these components integrated twice over proper time are finite at the horizon. This in turn means that the total distortion on an object is finite, and a small object can survive the tidal force when crossing the horizon. We will explain what the criterion for "small" is shortly.

The fact that the integrated tidal force remains finite is closely related to the fact that the metric is well defined and continuous at the horizon. It is also reminiscent of the singularity at the Cauchy horizon inside a charged or rotating black hole [8,9].

Let us first show that the curvature components are integrably finite. Consider the six dimensional black string case. Near the horizon, we have

$$|R_{i\tau j\tau}| = \frac{C_1}{r^2 - 1} = C_2 \frac{G^2}{W^2} = C_3 G^2 \tag{4.1}$$

where C_1, C_2, C_3 are finite constants. The second and third steps can be seen from equation (2.8). Next, let us define

$$\begin{aligned} &\sigma_0 \equiv \frac{1}{L} \int_0^L \sigma du, \\ &G_0(u) \equiv e^{\int_0^u \sigma_0 du} = e^{\sigma_0 u}, \quad U_0 \equiv -\int_u^{+\infty} \frac{du}{G_0^2} = -\frac{1}{2\sigma_0 G_0^2}, \\ &\eta_G \equiv \frac{G}{G_0}, \qquad \eta_U \equiv \frac{U}{U_0}. \end{aligned} \tag{4.2}$$

η_G is a periodic function with period L and therefore has a finite value bounded from below and above. From this, one can show that η_U is also bounded from below and above. So, $G^2 = \frac{C}{U}$ and

$$|R_{i\tau j\tau}| = \frac{C_4}{U} \tag{4.3}$$

Finally, let us notice that U is a null coordinate and $\frac{dU}{d\tau}$ is approximately a finite constant near the horizon. This leads to

$$|R_{i\tau j\tau}| = \frac{C_5}{\tau} \tag{4.4}$$

C, C_4, C_5 are all finite constants and τ is the proper time measured from the horizon.

Thus, the curvature components integrated twice over proper time are clearly finite at the horizon. This is also true for the five dimensional black string.

As an object falls into the black string, the tidal force gets bigger and bigger. We can divide this process into two periods. The first is when the stress caused by the tidal force is smaller than the maximal value that the object can resist. After the transition point, the stress caused by the tidal force exceeds this maximal value and we can think of the object as made of dust-like point particles that follow their own geodesics. The criterion of surviving the tidal force should be that the distortion has to be much smaller than the size of the object when it reaches the horizon.

At any time, there is a stress distribution within the object. The maximal stress T_{max} can be estimated as [10]:

$$T_{max} \sim \rho l^2 |R_{i\tau j\tau}| \tag{4.5}$$

where ρ is the mass density of the object and l is the linear size of the object. For a small object, the transition point will be close to the horizon. Given this and the fact that the curvature components are integrably finite, it is not difficult to see that the distortion will be small compared with the size of the object and therefore it will reach the horizon intact.

5. Discussion

We have seen that the addition of traveling waves to extremal black strings causes the horizon to become singular. This supports the “no hair conjecture” and reinforces the idea that solutions with regular horizons (having compact cross-sections) are characterized by only a few parameters. Nevertheless, the waves should not be discarded as totally unphysical. There are indications that these waves may still play a role in string theory. First, because of the null translational symmetry and special properties of the spatial geometry, these solutions are likely to be exact, and not receive α' corrections even though the curvature becomes large [4,11].

Second, in [5,6] it was shown that the waves traveling along an extremal black string affect the area of the event horizon as well as the distribution of momentum along the string.

Using recent results in string theory [12,13], it was also shown that one could reproduce the Bekenstein-Hawking entropy of this black string by counting states in weakly coupled string theory with the same charge and momentum distribution. This is true despite the fact that the curvature at the horizon diverges.

It should perhaps be noted that traveling waves can be added *inside* the horizon of a black string without causing the horizon to become singular [14]. In fact there are a large class of such modes, which may explain where information is stored inside the horizon.

We have considered the case where the direction along the black string is compact. If one considers the case where it is noncompact, then it is clear from the results in sections 2 and 3 that nonsingular waves are possible. One can simply choose $p(u)$ to be constant for large u or approach a constant sufficiently quickly. Then $\dot{\sigma}$ vanishes for large u fast enough so that all curvature components remain finite. However it is not clear whether these waves should be called classical hair. Since $u = t - z$, an asymptotic observer at fixed z cannot detect the wave at late times. In this sense, the "hair" is transitory. This is not a problem when z is periodic.

After this work was completed, [15] appeared which has some overlap with the results presented here.

Acknowledgments

It is a pleasure to thank D. Marolf for discussions. This work was supported in part by NSF Grant PHY95-07065.

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